Abstract—This research proposes an automatic parking system (APS) with a good maneuver performance for vehicle. The proposed configuration of the APS includes sensors information fusion, position estimation, path planning, and tracking algorithm. The on-vehicle verification demonstrated by means of the control of an electric power steering system for tracking the planned path. Parking space is measured up as the vehicle drives past with the aid of an ultrasonic sensor. And drivers can use a user interactive interface to select the parking space if it is big enough. Then steering movements take place on the basis of the pure pursuit tracking algorithm and the inertial navigation method. Simulation and experiment results show good performance in the verification of our automatic parking system under reverse parking and parallel parking cases, and the parking space requires just 1.5 times longer than vehicle length. Parking maneuvers will become safer and more efficient through the utilization of these advanced safety techniques for automatic parking.

Keywords—Automatic Parking System; Electric Power Steering; Path Planning; Tracking Control

I. INTRODUCTION

The automatic parking is one of the growing topics that aim to enhance the comfort and safety of driving. It can help drivers automatically drive the car in constrained environments where much attention and experience is required. The parking strategy is completed by means of coordinated control of the steering angle and taking into account the actual situation in the environment to ensure collision-free motion within the available space.

Several automobile research institutes and manufacturers are developing automatic parking systems. The group in INRIA has built up complete sensor based control architecture for an autonomous vehicle [1]. Their planning method is a model-based approach that decomposes the motion into a number of “parallel parking” series. By use of the ultrasonic sensors, the size of the free parking space is determined.

A commercial version of automatic parallel parking was introduced by Toyota Motor Corporation in Toyota Prius in 2004. Lexus also debuted a car, the 2007 LS, with an Advanced Parking Guidance System.

This paper presents a model-based technique that has the shortest planned path and tracking control algorithm to park automatically. Our approach consists of user interactive interface, ultrasonic data, path planning and path tracking for parallel and reverse (backing in) parking. Fig. 1 shows overall configuration of this automatic parking system.

II. EQUIPMENT

A. Sensors

Our system has been tested on our experimental vehicle platform, a Mitsubishi Savrin, is shown in Fig. 2. Sensors are equipped to measure the state of the experimental vehicle. A gyro is employed to measure the heading angle of the vehicle and an incremental wheel pulse transducer (WPT) is used for acquisition of wheel rotation and calculation of distance traveled. The implementation of tracking strategy on a dsPIC microprocessor makes this work a practical and feasible approach to commercial products.

Inertial navigation method based on instruments above enables the vehicle to estimate the vehicle positions. However, other equipments such as an ultrasonic sensor and a touch panel are also built in our system.

The ultrasonic sensor based on time-of-flight method is used to configure environment and the touch panel allows user to interact with the system.

Furthermore, a RTK-GPS provides high accurate and real-time X-Y state plane coordinate of the vehicle at a rate of 20Hz for the calibration of vehicle position calculated from sensor data and microprocessor.
B. Electric Power Steering

An EPS (electric power steering) composed of a motor, steering angle sensor and a power module is equipped into the steering wheel for implementation of automatic parking in this study.

The EPS is not commercially available and we develop it ourselves, its mechanism model is shown as Fig. 3. It is a typical column-type EPS system that consists of a torque sensor, an electric motor, a reduction gear, a column and a rack–pinion mechanism. Equations of motion are developed from Newton’s Law to facilitate our design of steering controller. The equations are shown below

\[
T_h - K_t (\theta_{sw} - \theta_{sc}) - B_{sw} \dot{\theta}_{sw} = J_{sw} \ddot{\theta}_{sw} \quad (1)
\]

\[
T_{\text{mn}} + T_f - B_{sc} \dot{\theta}_{sc} + K_r (\theta_{sw} - \theta_{sc}) - k_r (\theta_{sc} - \frac{x_r}{r}) = J_{sc} \ddot{\theta}_{sc} \quad (2)
\]

\[
\frac{k_r}{r} (\theta_{sc} - \frac{x_r}{r}) - F_r - b_r x_r = m_r \ddot{x}_r 
\]

where \(T_h\) is the instructional torque on the steering wheel from the a driver; \(K_t\) is the stiffness of the torsion bar; \(J_{sw}\) and \(B_{sw}\) are the inertia and the damping constant of the steering wheel; \(\theta_{sw}\) and \(\theta_{sc}\) are the steering wheel angle and the steering column angle respectively. \(T_{\text{mn}}\) and \(T_f\) are the electromagnetic drive and the friction torque on the steering column; \(J_{sc}\) and \(B_{sc}\) are the inertia and damping constant of the steering column. \(k_r\) is the stiffness between the rack and pinion; \(x_r\) is the dispacement of the rack; \(r\) is the stroke ratio. The angle of the pinion is equal to the column angle. \(F_r\) is the alignment force on the rack from the road wheel; \(m_r\) and \(b_r\) are the mass and the damping constant of the rack.

Since the EPS is equipped in the experimental vehicle, the steering wheel with a PID controller can be utilized to automatically steer the vehicle, as Fig.4. In addition, the PID controller parameters in the EPS loop are chosen by trials and errors with the aid of the EPS model as background knowledge.

III. AUTOMATIC PARKING SYSTEM

This approach has been implemented on microprocessor as Fig.1 and the core task could be divided into two procedures. The primary work is to push the “search available parking area button” on the input touch panel by the driver, and then microprocessor will calculate the vehicle travelled distance from the incremental wheel pulse transducer, and then convert the longitudinal distance as well as the lateral parking space by the ultrasonic sensor to build up an environment map in the memory of the microprocessor. The other work that driver has to do is to choose parking space again or adjust it for more considerations.

A. Path Planing

Fig. 5 shows the parking procedure of automatic parking system. After parking space is configured through the ultrasonic sensor, the next step is to decide the path from the original position to the final. This study focuses on the development of parking strategy for parking in the shortest parking space in continuous operation. Thus the minimum turning radius to fit the parking space is our first consideration. And the other route is determined from the

![Figure 5. Flow chart](image-url)

![Figure 4. Schematic diagram of the tracking subsystem.](image-url)

![Figure 3. Schematic diagram of the EPS system.](image-url)
starting position to the final position where the minimum turning radius can reach. Taking these requirements into consideration, a methodology based on geometry relation is developed and the turning radius in the first turn is shown as follows:

$$R_s = \frac{(l)^2 - 2R_{\text{min}}(w) + (w)^2}{2(w)}$$  \hspace{1cm} (4)$$

where $R_s$ is the turning radius of the first turn. $R_{\text{min}}$ is the minimum turning radius. $l$ is the longitudinal length and $w$ is the lateral width for parking.

After going through the route of first turning radius, the maximum steering angle is chosen to get into the smallest parking space and the parking procedure completes consequently as Fig. 6. The corresponding steering angle actuated by the EPS is shown in Fig. 7.

**B. Path tracking**

Path tracking is the process concerned with how to determine speed and steering settings at each instant of time in order for the vehicle to follow a certain path. A path tracking algorithm, pure pursuit, is adopted in this work. This approach is to calculate the curvature that will take the vehicle from its current position to a goal position [2]. A circle is then defined in such a way that it passes through both the goal point and the current vehicle position. Finally a control algorithm chooses a steering angle in relation to this circle. In fact, the vehicle changes its curvature by repeatedly fitting circular arcs of this kind, always pursuing the goal point forward, as Fig. 8.

Vehicle position is calculated simply based on the accumulation of traveled distance of wheel as equations (5) and (6) where $x_0$ and $y_0$ are the last positions, $x_0$ and $y_0$ are the current positions, $l$ is the distance traveled in the last time interval and $\psi$ is the yaw angle of gyro.

$$x_r = x_0 + l \cos(\psi)$$  \hspace{1cm} (5)$$

$$y_r = y_0 + l \sin(\psi)$$  \hspace{1cm} (6)$$

It is important to note that the description of the pure pursuit algorithm in Fig. 9 is shown in vehicle coordinates. The vehicle coordinate system is defined where the x-axis is in the forward direction of the vehicle and the y-axis forms a right-handed coordinate system. Therefore all coordinates used must first be transformed to vehicle coordinates in order for the algorithm to work properly. Luckily it is pretty straightforward to converts coordinates located in one system into its representation in another system. Let $(x_r, y_r)$ be the current position of the
vehicle, and \((x_{g_{v}}, y_{g_{v}})\) is the goal point to be converted into vehicle coordinates. Then

\[
y_{g_{v}} = (x_{g} - x_{r}) \cos(-\psi) - (y_{g} - y_{r}) \sin(-\psi) \] (7)
\[
x_{g_{v}} = (x_{g} - x_{r}) \cos(-\psi) + (y_{g} - y_{r}) \sin(-\psi) \] (8)

where \((x_{g_{v}}, y_{g_{v}})\) is the goal point in vehicle coordinates and \(\psi\) is the current vehicle heading. In the figure above \(D\) is defined to be the lookahead distance and \(\Delta y\) is the \(y\) offset of the goal point from the origin. The required curvature of the vehicle is computed by:

\[
\gamma = \frac{2\Delta y}{D^2} \] (9)

Given equations above, the tracking algorithm is developed and used to track a vehicle in the parallel parking and reverse parking situations.

One important factor to be implemented carefully is the lookahead distance because it determines the tracking stability and the precision of tracking position [3]. This study will show the influence of choice of lookahead distance by means of different experiment data.

IV. SIMULATIONS

The representative of vehicle parameters such as track width and minimum turning radius are used for the simulation in the MATLAB environment. Fig. 10 shows parallel and reverse parking simulation result for path planning. It shows that the two-turn model developed in the section III is feasible and the parking space requires just 1.5 times longer than vehicle length.

In addition, the tracking algorithm is also executed in the MATLAB environment and shows good performance. Fig. 11 shows that the maximum error in the parallel parking procedure occurs at the two-turn conjunction and its value equals 30cm. This is because the planned path changes from the first circle to the next and the demand for tracking abilities increases, and it also increases the deviation from planned path in the parking procedure. However, the error eventually converges to zero as the vehicle travel further to the final position. Thus this result remains within tolerance of parking system because the maximum error occurs at the two-turn conjunction where the error is not as critical as the final position.

V. EXPERIMENT RESULT

In spite of the success of simulation results, the verification on the vehicle platform is still necessary because some characteristics of vehicle dynamic might not be considered in our simulation model.

In the first place, parking space detection and environment configuration are measured by ultrasonic sensor while the vehicle travels at the speed of 10 km/hr. One parking lot available between two sedans is shown as Fig. 12. The width and the length of the parking space is measured and the errors is quite small.

Fig. 13 shows one of results in our test of automatic parking experiment. It shows both command and actual data of steering angles during parallel parking. The real and expected trajectory shows the errors.

Fig. 14 shows that different lookahead distance will contribute to the feature of tracking path. On the one hand, longer lookahead distances tend to converge to the path more gradually and have less oscillation; on the other hand, shorter lookahead distances decrease the deviation from the planned path so that real trajectory will be closer
to the expect trajectory. In our experiments, the lookahead distance equal to 200 is recommended to be adopted for future application.

Comparing vehicle positions from inertial navigation method with the RTK-GPS data shows that errors occur in application of inertial navigation method as Fig. 15. However, the results of reverse parking are also acceptable and further improvement of position estimation will make this automatic parking better.

Demonstrations of this automatic parking system are shown in Fig. 16 and Fig. 17. Both of them provide us more actual picture and make this work more persuasive.

VI. CONCLUSIONS AND DISCUSSIONS

One complete and circumspect parking system is proposed with the fusion of a user interactive interface, an environment configuration, a path planning and a path tracking. In this circumstance, a prototype of a parking system was developed and the experiments result of on-vehicle showed two competitive advantages. One is that this methodology just required a parking space about 1.5-time longer than the length of a car; the other is that the need for lateral starting distance only ranges between 0.5 and 2 m from the adjacent vehicle. This approach provides one system that can quickly fit all automobiles without any complex installation and calibration.

The position estimation from (5) and (6) at first glance seems to be a straightforward result. However, to acquire the correct travelled distance is more difficult than expectation. Many factors will contribute to the errors of traveled distance gotten from the wheel rotation. For example, the average of actual wheel diameters may differ from the nominal wheel diameter. The width of the wheel also makes the measurement of wheel rotation imprecise because the inner plane and the outer plane of the same wheel travel at different distance. Although a RTK GPS can provide us a solution of high accurate position measurement, it is too expensive for practical application. A solution combining with the available and other affordable sensors must be proposed in the near future for a feasible automatic parking system.
Finally, a feed-forward controller is the next step to design a more precise tracking algorithm since two primary steering angles are defined in this path strategy as Fig. 7. A feed-forward controller can be utilized to control the steering angle at the first place and the original pure pursuit algorithm is used to provide correction if necessary.

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