Car Trajectory Prediction in Image Processing and Control Manners

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Abstract—This paper studies car trajectory predictions desired in several active safety systems, such as automatic emergency braking (AEB) systems and lane keeping systems (LKS). In the former, the car trajectory is estimated such that objects detected within this trajectory in front of current host vehicle are taken into consideration of collision avoidance. In LKS, the trajectory predictor is utilized to evaluate a lateral displacement so as to keep the host car running within the selected lane by reducing this displacement. To accomplish the prediction task, a trajectory estimation strategy is newly proposed in a fusion manner. First, a road model—capturing road geometry characteristics and combined with a vehicle dynamics—is referred; it includes two parameters related to road curvature, which are both estimated in a nonlinear manner. An image processing mechanism is alternatively adopted as a compensating curvature estimator if the proposed observer was in transient response. After the evaluation of road curvature, the vehicle trajectory in future few seconds are estimated in a series of path positions for the car mass center. Experimental results show the proposed predictor guarantees at least 95% of estimation preciseness compared to a real path. Target sensing in AEB worked rapidly in real-world tests with the aid of the path estimator.

Keywords—road model; curvature estimation; trajectory prediction; nonlinear observer; image processing

I. INTRODUCTION

Car trajectory prediction has attracted large attention in past decades. Theoretically, the car trajectory can be mathematically described in two parameters—road curvature and its rate [1]; it was generally investigated with an aid of road curvature estimation. Solutions of the curvature observation problem were traditionally proposed in three manners: estimating road curvature based on a road model/vehicle dynamics/digital map. Published literatures [1-4] have constructed curvature observers in the first manner. Clothoid models were adopted to capture characteristics of curvature in these papers. It catches three kinds of path geometry—linear path geometry, quadratic path geometry, and cubic path geometry. Curvature and its rate in these three cases were observed and further applied to predict future car path. This contributes a benefit of low computation cost of realizing algorithms in embedded systems; however, the estimation response may be not in time enough. To modify this disadvantage, [5-8] alternatively solved the problem upon vehicle dynamics. High modeling preciseness of the car behavior brings fast convergence speed in estimations. However, this inevitably increases computation cost and complicates the strategy realization in embedded systems. Unlike the above two estimation methods, authors in [9] estimated a road curvature by using digital maps, which have been established in either image processing or lidar applications. Nevertheless, this manner has observation preciseness lower than others [1-8].

Motivated by two opposite drawbacks of methods in [1-4] and [5-8], we attempt to increase the convergence speed of estimations while holding acceptable realization cost simultaneously by combining strategies [1-4] and those [5-8]. Simultaneously, an image processing method for road curvature estimation is further treated as a compensator if the estimator worked failed. The addresses the lunching point of this research: the combination of ideas proposed in [1-4], [5-8], and [9]. Given the curvature information, the trajectory in future is evaluated in a series of path points.

II. STRATEGY DESIGN

A. Problem Statement

Sensing functions in the automatic emergency braking (AEB) system generally desire the car trajectory prediction for the object classification, as well as what illustrated in Fig. 1. Objects detected in front the host car can be classified into the ones lying within/without the running path. Only those within the future trajectory shall be focused and avoided in front-to-rear collisions. To fulfill this task, the problem stated

![Flowchart of a sensing mechanism in AEB.](flowchart.png)
here is defined as: how to predict the car trajectory in future with prediction preciousness larger than 95%. For the sake of convince, only path in future ten seconds is considered in prediction.

**B. Vehicle Dynamics Development**

The trajectory is constructed upon a road curvature and its variation rate. The former can be roughly obtained by mathematically modeling vehicle dynamics. Referring to [10] defines a dynamics:

\[
\dot{x} = Ax + B\delta f + B_0c
\]  

(1)

with \(x = [\beta \ r \ \Psi_L \ y_L]^T\), a sideslip angle \(\beta\), yaw rate \(r\), lateral displacement \(y_L\) from the center line of the host car with respect to yaw angle \(\Psi_L\), the corresponding look-ahead distance \(l_z\), steering angle \(\delta f\) of front wheels, the road curvature \(c\), and

\[
A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ v & l_z & v & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad B_0 = \begin{bmatrix} -v \\ -v \end{bmatrix}, \quad B_p = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

\[
a_{11} = -2\frac{c_r + c_f}{mv} a_{12} = -1 + 2\frac{(l_r c_r - l_f c_f)}{mv^2} \\
a_{21} = \frac{2(l_r c_r - l_f c_f)}{f} a_{22} = -\frac{2(l_r^2 c_r + l_f^2 c_f)}{fv},
\]

\[
c_r = c_{r0}v, \quad c_f = c_{f0}v, \quad b_1 = \frac{2c_r}{mv}, \quad b_2 = \frac{2c_f}{f}
\]

Please refer to Fig. 11 in [10] for more details of parameter definitions. The road curvature is thus computed as

\[
c = B_p^T(\dot{x} - Ax - B\delta f)
\]  

(2)

where this values commonly features measurement noise in real-world applications if it was computed upon sensor data. This redefines the measured curvature \(c_d \equiv c + d\) with the disturbance \(d\), coming from car skidding in driving.

**C. Road Model Development**

The curvature rate \(c_1 \equiv dc(t)/dt\) in which \(k(t)\) represents the length of paths a car passed by—is estimated under the road model in [1]. It captures road curvature characteristics with three kinds of road geometry: 1) linear path segments; 2) cubic path segments; and 3) quadratic path segments. Please refer to Fig. 2. The clothoid model [1-4] is mathematically represented by

\[
c(t) = c_0 + c_1l(t)
\]  

(3)

where \(c(t)\) denotes the road curvature at time \(t\), constant \(c_0\) means the road curvature in beginning of car driving, and \(l(t)\) represents the path length a car passed by. Notice that substituting \(c_0 = c_1 = 0\) into (3) derives the road curvature \(c = 0\) for a linear path. Setting \(c_1 \neq 0\) and \(c_0 = 0\) in (3) next computes the road curvature \(c(t) = c_0\) in the quadratic segment. Last, the road curvature of the cubic segment is represented as (3) with \(c_0 \neq 0\) and \(c_1 \neq 0\).

Differentiating (3) with respect to time \(t\) generates

\[
\dot{c}(t) = c_1v, \quad (4)
\]

\[
c_1 = 0 \quad (5)
\]

\[
y = c \quad (6)
\]

with \(v\) being the vehicle speed and \(y\) denoting a system output. Motivated by the existence of \(d\), (6) is rewrote as \(y = c_d\). Equations (4)-(5) along with \(y = c_d\) are treated as the dynamics of (2).

**D. Curvature Rate Estimation**

This section aims at estimating \(c_1\) upon (4)-(5) with \(y = c_d\). The observer is correspondingly proposed in a nonlinear manner with \(\dot{\hat{c}}_1\) estimating \(c/c_1\) and \(k\) defining an observer gain. Defined \(e_1 \equiv c - \hat{\hat{c}}_1\) and \(e_2 \equiv c_1 - \hat{\hat{c}}_1\) with the consideration of this observer, one further derives

\[
\dot{e}_1 = ve_2 - ke_1 - kd
\]  

(7)

\[
\dot{e}_2 = -ve_1 - vd
\]  

(8)

That is,

\[
\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -k & v \\ -v & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} - \begin{bmatrix} k \\ v \end{bmatrix} d
\]  

(9)

where \(v\) is supposed to be constant. System (9) guarantees bounded-input-bounded-output stability if \(k\) is selected to hold the stable \(\begin{bmatrix} -k & v \\ -v & 0 \end{bmatrix}\) under the norm-bounded \(d\). Performing an analysis of steady-state errors of (9) indicates that (8) is globally asymptotically stable although a disturbed output was injected into the nonlinear observer. Studying the convergence of \(e_1/e_2/\hat{\hat{e}}_{k+1} - \hat{\hat{e}}_k\) determines whether the nonlinear observer and (8) are in transient/steady response.

![Figure 2. Path types.](image-url)
E. Road Curvature Filtering

The measurement output $c_d$ commonly features high-frequency noise. For example, Fig. 3(b) illustrates a power spectral density analysis of a noisy road curvature radius—denoted by $1/c$—instead of the road curvature for convenience. In Fig. 3(a), the yellow line represents the original curvature radius computed based on (2) and $1/c$. It is obvious that the original signal reveals serious oscillation, a high-frequency noise, coming from the road curvature signal around the radius 31 m. Results in Fig. 3(b) then verify this point. Note that -30 m in Fig. 3 means that the vehicle ran around a circle with the radius 30 m anticlockwise. Predicting trajectory without noise filtering indeed decreases the prediction preciseness; therefore, the oscillation should be suppressed. The curvature radius with noise filtering is denoted by the red line in Fig. 3(a). Methods of oscillation suppression are next explained in details.

A discrete-time filter is established under a supposition: the sampling time $T$ is small enough such that the sampled signal is quite the same as current sample. That is, $c_{k+1} = c_k$ with $c_k$ denoting the curvature sampled at time $kT$. The filter is corresponding proposed as a discrete-time observer under the definition that $\dot{c}_k$ estimates $c_k$ and $\hat{L}$ is an observer gain included in this observer. Performing stability analysis under the proposed observer with $e_k \equiv c_k - \hat{c}_k$ gives that the error dynamics of $e_k$ would be globally asymptotically stable if $|1 - L| < 1$. Note that the smaller $|L|$ is, the more suppression of the oscillation one has.

To this end, we have proposed: i) the nonlinear observer estimates $c_1$, and ii) the discrete-time estimator observes the clear road curvature. These two estimations are further applied to predict the desired trajectory upon the strategy in [3]. Under a practical concern of compensation design, there is a problem worth of further attention: how to get precious estimations if either the nonlinear observer or the discrete-time estimator is in transient behavior. The solution is next designed so that the road curvature is computed as a constant via an image analysis and $c_1$ is treated as zero.

F. Compensation Design in Image Processing

Current image-processing part is proposed with an aid of lane detection; it as concluded as a flowchart shown in Fig. 4. In the beginning, DSP captures video from CCD/CMOS camera and detects the lane mark. Road curvature is next evaluated based on the detected lane marks. This strategy would be alternatively to estimate the road curvature if the proposed observer was in transient response.

To speed up the detection rate, image processor sets a region of interest (ROI) and vision points. DSP scans the image from its bottom and then finds the initial image row of lane mark to determine the ROI. The processor further finds the pixel point of lane mark in each image row. The vision point of lane mark is computed according to pixel point of lane mark; it could be treated as a criterion of lane detection. Road curvature is finally evaluated based on lane marks. To illustrate, Fig. 5 shows a road curvature calculation model. Given the detected lane mark, the road model can be captured by (10)-(11) consisting of parameters defined in Fig. 5. Finally, the road curvature $c$ is further calculated as (12).

$$y_L = k \cdot y_d^2 + m \cdot y_d + b$$  \hspace{1cm} (10)

$$\varepsilon_L = 2 \cdot k \cdot y_d + m$$  \hspace{1cm} (11)

$$c = \frac{2k}{(1+(2-k\cdot y_d+m))^2}$$  \hspace{1cm} (12)

where $k$, $m$, and $b$ are the model parameters, $\varepsilon_L$ is the slope of the route calculated from (10), $y_L$ means lateral displacement, and $y_d$ is the current displacement.

G. Compensation Design in Image Processing

The trajectory in future ten seconds is derived based on the estimations of $c_1$ and $c$ while we have made a supposition—the yaw rate and vehicle speed both remain invariant during the considered time period. Figure 6 illustrates the trajectory satisfying the previous supposition. Referring to [3] defines $\Delta \theta (t) = \int_0^t c(t) dt = c_d \frac{1}{2} c_s l^2$ in Fig. 6. Given the vehicle speed $v$, we have $dl = vdt$; in other words, $l = vt$. Along with $x(l) = v \sin(\Delta \theta)$ and $y(l) = v \cos(\Delta \theta)$ at the position $p$, the car path is

![Triaxial Multimodal Power Spectral Density Estimate](image)

Figure 3. An example of power spectral density analysis, (a) road curvature radius, and (b) power spectral density.

![Camera image processing](image)

Figure 4. Lane detection flowchart.
Figure 5. Image road curvature calculation model.

Figure 6. Car trajectory calculation mode

\[
x(t) = \int_0^t \dot{x}(\tau)d\tau = \int_0^t v_0t\sin\left(c_0v_0^2t^2 + \frac{1}{2}c_1v_0^2r^2\right)\,d\tau
\]

\[
y(t) = \int_0^t \dot{y}(\tau)d\tau = \int_0^t v_0t\cos\left(c_0v_0^2t^2 + \frac{1}{2}c_1v_0^2r^2\right)\,d\tau
\]

with \( t \in [0,10] \).

III. EXPERIMENTAL RESULTS

Two experiments were conducted: one is offline performed to evaluate the preciseness of path prediction while the other is online conducted to show the advantage of combining it with AEB.

A. Offline Experiment

This section shows the experimental results conducted around the Frick Park in Pittsburgh, PA, U.S.A., as shown in Fig. 7. Predicting preciseness is evaluated by comparing the car trajectory computed offline and the real one in a series of GPS points captured online, as the yellow points in Fig. 7.

For the sake of convenience, the path segment pointed out in Fig. 7 was only taken into consideration of evaluation. Performing steps in section II.H generates the trajectory as the green line displayed in Fig. 8, in which the blue line denotes the path of the mass center of the host car. Moreover, the two red lines state the trajectory boundaries. Given a constant width of the trajectory, prediction preciseness is defined as the coverage rate between the real area car passed by and that in prediction. In Fig. 8, the coverage area is 73.495 m\(^2\) and the real path area is 76.595 m\(^2\) so that the estimating preciseness is 73.495/76.595 \* 100% = 95.953%. This indicates our method has guaranteed the requirement in Section II.A in the first experiment.

Figure 7. Car trajectory in experiments.

Figure 8. Comparison between the predicted and real vehicle paths.

Similar to the experiment conducted in U.S.A., another offline experiment was performed with the vehicle in Fig. 9(a) in the proving ground of Automotive Research & Testing Center in Taiwan, as displayed in Fig. 9(b). The proposed strategy was verified by driving this car in curve roads with curvature radius being 10 m/ 20 m/ 30 m/ 40 m/ 50 m/ 60 m (see Fig. 10) and car speed lying within [10, 50] (km/h). Table I lists the estimating preciseness for this test.

![Experimental setting, (a) vehicle and (b) proving ground.](image)

**Figure 9.** Experimental setting, (a) vehicle and (b) proving ground.

**Figure 10.** Driving upon a curve line.

**TABLE I. EXPERIMENTAL RESULTS**

<table>
<thead>
<tr>
<th>Curvature radius</th>
<th>Estimating preciseness</th>
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<tbody>
<tr>
<td>10 m</td>
<td>98.2%</td>
</tr>
<tr>
<td>20 m</td>
<td>97.5%</td>
</tr>
<tr>
<td>30 m</td>
<td>98.6%</td>
</tr>
<tr>
<td>40 m</td>
<td>98.4%</td>
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<tr>
<td>50 m</td>
<td>97.8%</td>
</tr>
<tr>
<td>60 m</td>
<td>98.1%</td>
</tr>
</tbody>
</table>
Notice that the estimating precision was evaluated in the same way as mentioned previously. Moreover, Table I implies that our method has satisfied the requirement in Section II.A.

B. Online Experiment

The proposed mechanism was realized in an AEB system, equipped on the vehicle of Fig. 9(a), by following the flowchart in Fig. 1. The vehicle had millimeter-wave radars installed in the front bumper and a camera fixed on a windscreen; it ran on an inter-urban road with its speed higher than 80 km/h in the on-line experiment. Parts of experimental results are showed in Figs. 11. Figure 11(a) illustrates curvature evaluation results in the image process while the observer was in transient response. The curve line states the lane detection fitted to the lane mark. Moreover, the “IND” shows the road curvature calculated from DSP; it was alternatively adopted as a compensating curvature estimator. Figure 11(b) displays the target focused by AEB system, and Fig. 11(c) illustrates original sensing results come from the radar. Compared Fig. 11(c) with Fig. 11(b), the radar has cached four objects in that moment; however, three among them were guardrail objects. They should be filtered by the sensing system in AEB so that this system would not activate braking mistakenly. This has been accomplished by adding the proposed trajectory prediction in the AEB system. Only objects within the predicted trajectory were considered as focused candidates in AEB. The nearest one of them was treated as the focused target, as well as what displayed in Fig. 11(b). This indicates that adding the proposed strategy in the sensing mechanism successfully helps AEB system filter unnecessary objects, especially when one drove in a curve.

IV. CONCLUSION

This research has investigated a car trajectory prediction desired in AEB/LKS systems based on a road curvature estimation in a fusion sense. The predictor is designed in both control and image processing manners. Design in the former ensures that the path predictor guarantees estimation preciousness larger than 95%. Moreover, applying a clothoid model in the curvature observation reduces the complication in strategy realization on embedded systems. Also, combining a vehicle dynamics with this model further ensures the curvature estimation works in time. Additionally, according to a compensation requirement in real-world applications, a curvature estimator via image analysis is newly proposed to fit this requirement. Finally, experimental results reveal that the construction of the path estimation simplifies a target capturing in AEBs.

REFERENCES


AUTHORS’ BACKGROUND

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<td>Automatic emergency braking system</td>
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